

**PROBLEM SET 11**

1. French problem 6-9.
2. French problem 6-15(a).
3. French problem 8-9. Note that  $\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$  for the sodium atoms in the vapor, where  $T$  is the temperature in degrees Kelvin ( $^{\circ}\text{K}$ ), and  $k$  is Boltzmann's constant,  $1.38 \times 10^{-23} \text{ J}/^{\circ}\text{K}$ .
4. French problem 8-12.
5. A tank is filled with water to a height  $H$ . A hole is punched in its wall a depth  $h$  below the surface of the water.
  - a. Find the horizontal distance from the bottom of the tank that the stream of water hits the ground.
  - b. Could a hole punched at a different depth produce a stream with the same horizontal range? If so, at what depth?
6. Consider the stagnant air at the front edge of an airplane wing and the air rushing over a wing surface at speed  $v$ . Find the greatest possible value for  $v$  in streamline flow, using Bernoulli's equation and assuming that air is incompressible. Take the density of air to be  $1.2 \times 10^{-3} \text{ g/cm}^3$ . Compare this numerical result with the speed of sound, 340 m/sec.
7. Verify by explicit computation in a Cartesian coordinate system that

$$\nabla \times (\nabla f(x, y, z)) = 0$$

8.

- (a.) Consider the function  $f(x, y, z) \equiv x^2 + y^2 - z^2$ . At the point  $(x, y, z) = (3, 4, 5)$ , find the *direction* of a vector  $d\mathbf{s}$  (of small fixed length) such that  $df/|d\mathbf{s}|$  is a maximum.
- (b.) Consider the surface  $z(x, y) = \sqrt{x^2 + y^2}$ . At the point  $(x, y, z) = (3, 4, 5)$ , find the *direction* of a vector  $d\mathbf{u}$  (of small fixed length) which is normal to this surface.

9. A fluid has a velocity field

$$\mathbf{v}(x, y, z, t) = (\hat{\mathbf{y}}x - \hat{\mathbf{x}}y)\omega(t)$$

where  $\omega$  is some function of time  $t$ .

- (a.) Prove that the fluid density  $\rho(x, y, z, t)$  satisfies

$$y \frac{\partial \rho}{\partial x} - x \frac{\partial \rho}{\partial y} = \frac{1}{\omega} \frac{\partial \rho}{\partial t}$$

- (b.) Show that the angular velocity of the fluid about the origin, evaluated at an arbitrary point, is half of  $\nabla \times \mathbf{v}$  evaluated at the same point.
- (c.) If  $\omega(t) = \omega_0 = \text{constant}$ , prove that

$$\frac{d\mathbf{v}}{dt} = -\mathbf{r}\omega_0^2$$

where  $\mathbf{r} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y$ , and  $d\mathbf{v}/dt$  is the time rate of change of the velocity of an element of fluid that is *temporarily* at  $(x, y, z)$  at time  $t$ .